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On the short-distance constraints from $K_{L,S} \to \mu^+\mu^-$

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Abstract: Motivated by new precise results on several K_L decays, sensitive to the $K_L \to \gamma\gamma$ form factor, we present a new analysis of the $K_L \to \mu^+\mu^-$ long-distance amplitude based on the semi-phenomenological approach of ref. [1]. Particular attention is devoted to the evaluation of the uncertainties of this method and to the comparison with alternative approaches. Our main result is a conservative upper bound of 2.5×10^{-9} on $B(K_L \to \mu^+\mu^-)_{\rm short}$, which is compatible with the SM expectation and which provides significant constraints on new-physics scenarios. The possibility to extract an independent short-distance information from future searches on $K_S \to \mu^+\mu^-$ is also briefly discussed.

Keywords: Rare Decays, Kaon Physics, Chiral Lagrangians.

Contents

1. Introduction

The rare decays $K_{L,S} \to \mu^+\mu^-$ are a very useful source of information on the short-distance structure of $\Delta S = 1$ flavor-changing neutral-current (FCNC) transitions. In both cases the decay amplitude is not dominated by the clean short-distance contribution; however, long- and short-distance components are comparable in size. As a result, even with a limited knowledge of the long-distance component, it is possible to extract significant constraints on the short-distance part.

At present the $K_L \to \mu^+\mu^-$ decay is particularly interesting in this perspective because of the very precise determination of its decay rate [2]. Here the key issue to extract bounds on the short-distance amplitude is the theoretical control of the $K_L \to \gamma\gamma$ form factor with off-shell photons. An effective strategy to reach this goal is the combination of phenomenological constraints from $K_L \to \gamma\gamma$, $K_L \to \gamma\ell^+\ell^-$ and $K_L \to \mu^+\mu^+e^+e^-$ data, with theoretical constraints from chiral symmetry, large N_C, and perturbative QCD, proposed in ref. [1]. As far as the phenomenological constraints are concerned, the situation has substantially improved in the last few years thanks to precise results on all the K_L Dalitz modes [3, 4, 5] and, to a minor extent, because of the improvements on $K_L \to \gamma\gamma$ [6, 7]. On the theory side, alternative and/or complementary approaches about the high-energy constraints on the $K_L \to \gamma\gamma$ form factor have been discussed in ref. [8]–[11].

The main purpose of this paper is a new analysis of the $K_L \to \mu^+\mu^-$ two-photon dispersive amplitude, taking into account these new experimental and theoretical developments. In particular, as far as the theoretical constraints are concerned, we shall quantify the uncertainty of the method in ref. [1] using a more general parameterization of the form factor. Moreover, we shall discuss in detail differences and similarities between this approach and the one of ref. [8, 9]. As a result of this analysis, we find a conservative upper bound of 2.5×10^{-9} on $B(K_L \to \mu^+\mu^-)_{\rm short}$, which is compatible with the SM expectation, $B(K_L \to \mu^+\mu^-)^{\rm SM}_{\rm short} \approx 0.9 \times 10^{-9}$, and which provides significant constraints on new-physics scenarios.

On the experimental side, the situation of the $K_S \to \mu^+\mu^-$ decay is very different with respect to the $K_L \to \mu^+\mu^-$ one: this process has not been observed yet, and present experimental limits are still very far from the SM expectation, $B(K_S \to \mu^+\mu^-)^{\rm SM} \approx 5 \times 10^{-12}$ [12]. On the theory side, the $K_S \to \mu^+\mu^-$ amplitude is particularly interesting since its short-distance component is dominated by the CP-violating part of the $s \to d\ell^+\ell^-$ amplitude, which is very sensitive to new physics and which is poorly constrained so far. As we shall show, future searches of $K_S \to \mu^+\mu^-$ in the 10^{-11} range, could provide significant constraints on several consistent new-physics scenarios.

The plan of the paper is as follows: in section 2 we present the general decomposition of $K_L \to \mu^+\mu^-$ amplitude and branching ratio, and briefly review the structure of the short-distance component. In section 3 we discuss the evaluation of the low-energy coupling $\chi_{\gamma\gamma}$, which encodes the information on the off-shell $K_L \to \gamma\gamma$ form factor; using this estimate, in section 4 we analyse the present short-distance constraints from $K_L \to \mu^+\mu^-$. Section 5 is devoted to $K_S \to \mu^+\mu^-$. The results are summarized in the Conclusions.

2. Decomposition of the $K_L \to \mu^+\mu^-$ amplitude

2.1 General structure and experimental constraints

Within the Standard Model the $K_L \to \mu^+\mu^-$ decay is mediated by a CP-conserving s-wave amplitude which can simply be written as

$$\mathcal{A}(K_L \to \mu^+\mu^-) = (A_{\gamma\gamma} + {\rm Re}\, A_{\rm short})\, \bar\mu\gamma_5\mu\,, \tag{2.1}$$

where $A_{\gamma\gamma}$ denotes the complex long-distance contribution generated by the two-photon exchange and ${\rm Re}\, A_{\rm short}$ the real amplitude of short-distance origin induced by Z-penguin and box diagrams (see figure 1). The long-distance amplitude has a large absorptive part (${\rm Im}\, A_{\gamma\gamma}$) which is completely dominated by the two-photon cut and provides the dominant contribution to the total $K_L \to \mu^+\mu^-$ rate.[1] It is then convenient to normalize $\Gamma(K_L \to \mu^+\mu^-)$ to $\Gamma(K_L \to \gamma\gamma)$ and decompose it as follows [8, 10]

$$\Gamma(K_L \to \mu^+\mu^-) = \frac{2\alpha_{\rm em}^2 r_\mu \beta_\mu}{\pi^2}\,[R_{\rm abs} + R_{\rm disp}]\,\Gamma(K_L \to \gamma\gamma)\,, \tag{2.2}$$

[1] In principle, the absorptive amplitude receives additional contributions from real intermediate states other than two photons, such as two- and three-pion cuts, but these are completely negligible [13].

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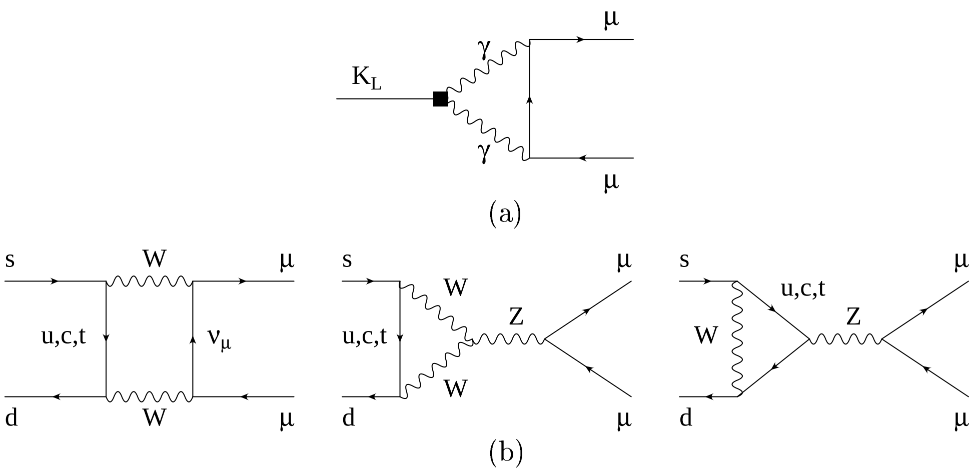

Figure 1: Leading long-distance (a) and short-distance (b) contributions to $K_L \to \mu^+\mu^-$.

$$R_{\rm abs} = ({\rm Im}\, C_{\gamma\gamma})^2 = \left[\frac{\pi}{2\beta_\mu} \ln \frac{1-\beta_\mu}{1+\beta_\mu}\right]^2 , \tag{2.3}$$

$$R_{\rm disp} = \left[\chi(\mu) - \frac{5}{2} + \frac{3}{2} \ln\left(\frac{m_\mu^2}{\mu^2}\right) + {\rm Re}\, C_{\gamma\gamma}\right]^2 , \tag{2.4}$$

where $r_\mu = m_\mu^2/m_K^2$, $\beta_\mu = \sqrt{1-4r_\mu}$ and

$$C_{\gamma\gamma} = \frac{1}{\beta_\mu}\left[{\rm Li}_2\left(\frac{\beta_\mu - 1}{\beta_\mu + 1}\right) + \frac{\pi^2}{3} + \frac{1}{4}\ln^2\left(\frac{\beta_\mu - 1}{\beta_\mu + 1}\right)\right] . \tag{2.5}$$

The real coupling $\chi(\mu) = \chi_{\rm short} + \chi_{\gamma\gamma}(\mu)$ encodes both short- and long-distance contributions: $\chi_{\rm short}$ denotes the genuine short-distance (scale-independent) contribution of the diagrams in figure 1b; $\chi_{\gamma\gamma}(\mu)$ is a low-energy effective coupling — determined by the behavior of the $K_L \to \gamma\gamma$ form factor outside the physical region — which compensates the scale dependence of the diagram in figure 1a (computed with a point-like form factor and regularized in the $\overline{MS}$ scheme).

The smallness of the total dispersive amplitude is well established thanks to precise experimental results on both $\Gamma(K_L \to \mu^+\mu^-)$ and $\Gamma(K_L \to \gamma\gamma)$. As pointed out by Littenberg [16], we can minimize the experimental error on the ratio $\Gamma_L(\mu^+\mu^-)/\Gamma_L(\gamma\gamma)$ decomposing it as the product of $\Gamma_L(\mu^+\mu^-)/\Gamma_L(\pi^+\pi^-) = (3.48 \pm 0.05) \times 10^{-6}$ [2] times $\Gamma_L(\pi^+\pi^-)/\Gamma_L(\gamma\gamma)$. The latter can in turn be determined as follows

$$\frac{\Gamma_L(\gamma\gamma)}{\Gamma_L(\pi^+\pi^-)} = \left\{ \begin{array}{ll} \frac{\Gamma_L(\gamma\gamma)}{\Gamma_L(\pi^0\pi^0)} \times \frac{\Gamma_L(\pi^0\pi^0)}{\Gamma_L(\pi^+\pi^-)} & = 0.2798 \pm 0.0044 \ [16] \\ \\ \frac{\Gamma_L(\gamma\gamma)}{\Gamma_L(3\pi^0)} \times \frac{\Gamma_L(3\pi^0)}{\Gamma_L(\pi^+\pi^-)} & = 0.2836 \pm 0.0060 \end{array} \right\} = 0.2811 \pm 0.0035 , \tag{2.6}$$

where, in the second case, we have used $\Gamma_L(\gamma\gamma)/\Gamma_L(3\pi^0) = (2.805 \pm 0.018) \times 10^{-3}$ from the average of NA48 [6] and KLOE [7] recent results and $\Gamma_L(\pi^+\pi^-)/\Gamma_L(3\pi^0) = (9.89 \pm$

$0.20) \times 10^{-3}$ from PDG [17]. The combination of the two ratios leads to

$$\frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K_L \to \gamma\gamma)} = (1.238 \pm 0.024) \times 10^{-5} \,. \tag{2.7}$$

According to eqs. (2.2)–(2.4) this implies

$$R_{\rm disp} = [\chi_{\gamma\gamma}(M_\rho) + \chi_{\rm short} - 5.12]^2 = 0.98 \pm 0.55 \qquad \text{or } R_{\rm disp} < 1.7 \text{ at } 90\% \text{ CL} \,, \tag{2.8}$$

to be compared with $R_{\rm abs} = 27.14$.

2.2 The short-distance component

In order to discuss the short-distance component of the $K_L \to \mu^+\mu^-$ amplitude, we need to specify more clearly how we separate short and long distances. Since we are interested in constraints on possible extensions of the SM, we choose as reference short-distance scale the scale of electroweak symmetry breaking. It is then natural to denote as SM short-distance amplitude only the $\mathcal{O}(G_F^2)$ contribution arising by the second-order electroweak interactions shown in figure 1*b*: these diagrams lead to a scale- and scheme-independent result (up to $\mathcal{O}(G_F^2\alpha_{\rm em})$ corrections) which is largely dominated by the top-quark contribution and which receives small QCD corrections. This component of the amplitude can be predicted with excellent accuracy in terms of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements V_{ij}. Following the notation of ref. [14], we find

$$\text{Re}\, A^{\rm SM}_{\rm short} = -\frac{G_F \alpha_{\rm em}(M_Z)}{\pi \sin^2\theta_W}\sqrt{2} m_\mu F_K \left[\text{Re}(V_{ts}^* V_{td}) Y(x_t) + \text{Re}(V_{cs}^* V_{cd}) Y_{NL}\right] . \tag{2.9}$$

Employing the decomposition (2.2)–(2.4) this leads to

$$\chi^{\rm SM}_{\rm short} = \kappa \left[\frac{-\,\text{Re}(V_{ts}^* V_{td}) Y(x_t) - \text{Re}(V_{cs}^* V_{cd}) Y_{NL}}{4 \times 10^{-4}}\right] = \kappa(1.11 - 0.92\bar\rho) \,, \tag{2.10}$$

$$|\kappa| = 4 \times 10^{-4} \left[\frac{m_K}{16\pi \Gamma(K_L \to \gamma\gamma)}\right]^{1/2} \frac{\sqrt{2} G_F m_K F_K \alpha_{\rm em}(M_Z)}{\sin^2\theta_W \alpha_{\rm em}} = 1.96 \,, \tag{2.11}$$

where the second identity in (2.10) has been derived using the modified Wolfenstein parameterization of the CKM matrix [15] and the numerical estimates of $|V_{cb}|$ and $|V_{us}|$ from ref. [15] (the two coefficients in eq. (2.10) are affected by a $\sim 5\%$ error, mainly due to the uncertainty on $|V_{cb}|$ and Y_{NL}). The normalization of eq. (2.9) is conveniently expressed in terms of $G_F\alpha_{\rm em}$, rather than G_F^2. For this reason, in eq. (2.9) the value of $\alpha_{\rm em}$ refers to the electromagnetic coupling renormalized at the electroweak scale [$\alpha_{\rm em}(M_Z) = 1/128$], while in the long-distance (two-photon) part of the amplitude we use $\alpha_{\rm em} = 1/137.036$.

It must be stressed that this separation of short and long distances is an arbitrary choice. From the point of view of an effective low-energy theory with cut-off around 1 GeV, such as Chiral Perturbation Theory, also the two-photon amplitude with highly virtual photons leads to a non-negligible 'short-distance term' calculable by means of perturbative QCD [18, 19, 20]. In our approach, this contribution is encoded in $\chi_{\gamma\gamma}(\mu)$. We prefer not to separate perturbative and non-perturbative components of $\chi_{\gamma\gamma}(\mu)$ since: i) this separation in not scale and scheme independent (contrary to the separation between $\chi_{\rm short}$ and $\chi_{\gamma\gamma}$); ii) the perturbative contribution to $\chi_{\gamma\gamma}$ is dominated by energy scales around or below the charm mass [20], therefore is not sensitive to possible non-standard dynamics.

With the exception of rather exotic scenarios, all the non-standard contributions to the $K_L \to \mu^+\mu^-$ amplitude can be absorbed into a redefinition of $\chi_{\rm short} \doteq \chi(\mu) - \chi_{\gamma\gamma}(\mu)$. This happens, in particular, in the wide class of models where the dominant non-standard effects can be encoded in effective FCNC couplings of the Z boson: a well-motivated framework [21, 22] with renewed phenomenological interest [23]. Following the notation of ref. [22], here one can write

$$\chi_{\rm short} \;=\; \chi_{\rm short}^{\rm SM} - \kappa \frac{{\rm Re}\left(Z_{ds} - Z_{ds}^{\rm SM}\right)}{4 \times 10^{-4}} \,, \tag{2.12}$$

where $Z_{ds}^{\rm SM} = C_0(x_t) V_{ts}^* V_{td}$ and $C_0(x_t) = 0.79$.

3. Theoretical estimate of $\chi_{\gamma\gamma}(\mu)$

3.1 The $K_L \to \gamma\gamma$ form factor

The necessary ingredient to estimate the low-energy coupling $\chi_{\gamma\gamma}(\mu)$ is the $K_L \to \gamma\gamma$ form factor, $f(q_1^2, q_2^2)$, defined by the four-point Green's function

$$\int d^4x e^{iq_1 x} \int d^4y e^{iq_2 y} \langle 0 | T\left\{ J^\mu_{\rm em}(x) J^\nu_{\rm em}(y) \mathcal{L}_{\Delta S=1}(0) \right\} | K_L \rangle \propto \epsilon^{\mu\nu\rho\sigma} q_{1\rho} q_{2\sigma} f(q_1^2, q_2^2) \tag{3.1}$$

and normalized such that $f(0,0) = 1$. In terms of this form factor the $K_L \to \gamma\gamma$ vertex of figure 1 is written as

$$\mathcal{A}\left[K_L \to \gamma(\varepsilon_1, q_1)\gamma(\varepsilon_2, q_2)\right] = i \frac{\mathcal{N}}{m_K} f(q_1^2, q_2^2) \epsilon_{\mu\nu\rho\sigma} \varepsilon_1^\mu \varepsilon_2^\nu q_1^\rho q_2^\sigma \,, \tag{3.2}$$

with $\mathcal{N} = [64\pi\Gamma(K_L \to \gamma\gamma)/m_K]^{1/2} = 1.74 \times 10^{-9}$, and

$$\chi_{\gamma\gamma}(\mu) = \left\{ \frac{32\pi^2 i}{q^2} \int \frac{d^d k}{(4\pi)^d} \frac{[q^2 k^2 - (q\cdot k)^2][f(k^2, (k-q)^2) - 1]}{k^2 (k-q)^2 [(k-p)^2 - m_\mu^2]} \right\}_{\overline{MS}} \tag{3.3}$$

with $q^2 = m_K^2$ and $p^2 = m_\mu^2$. As can be understood from eq. (3.3), the leading contribution to $\chi_{\gamma\gamma}(\mu)$ in the chiral expansion (or in the limit where we neglect external momenta) is governed by $f(-k_E^2, -k_E^2)/k_E^2$, with $k_E^2 \in [0, \infty]$.

The structure of $f(q_1^2, q_2^2)$ has been investigated by several authors (see e.g. ref. [1, 8, 9, 10, 11, 24]). The general properties dictated by QED and QCD can be summarized as follows:

i. it is symmetric under the exchange $q_1^2 \leftrightarrow q_2^2$;

ii. it is analytic but for cuts and poles in the region $q_{1,2}^2 > 4m_\pi^2$, corresponding to the physical thresholds in the photon propagators;

iii. the high-energy behavior in the euclidean region is given by $f(-k_E^2, -k_E^2) \sim 1/k_E^2$ (up to logarithmic corrections).

Note that not all the parameterizations proposed in the literature satisfy these conditions. In particular, the properties ii. and iii. are not fulfilled by some of the parameterizations of ref. [11]: this is one of the main reasons why we do not agree with the conclusions of this work.

In addition to these general properties, a systematic and powerful approach which leads to tight constraints on the structure of $f(q_1^2, q_2^2)$ is provided by the large N_C expansion [8]. At the lowest order in $1/N_C$ we expect

$$f(q_1^2, q_2^2) = \sum_i \left[1 + \alpha_i \left(\frac{q_1^2}{q_1^2 - M_i^2} + \frac{q_2^2}{q_2^2 - M_i^2} \right) + \beta_i \frac{q_1^2 q_2^2}{(q_1^2 - M_i^2)(q_2^2 - M_i^2)} \right], \tag{3.4}$$

with

$$\sum_i [1 + 2\alpha_i + \beta_i] = 0\,, \tag{3.5}$$

where the sum extends over an infinite series of (infinitely narrow) vector-meson resonances and the sum-rule (3.5) follows from the $k_E^2 \to \infty$ condition [8]. This type of structure is certainly not exact point-by-point in q^2, but it has been shown to provide an excellent tool to evaluate dispersive integrals of the type (3.3), even when the sum is truncated to the first non-trivial term (see e.g. ref. [8, 25]). The truncation to the lowest vector-meson resonance (the ρ meson) of eq. (3.4) corresponds to the DIP ansatz [1] (with $\beta = -1 - 2\alpha$) and also to the choice of ref. [9]. Within such approximation, the form factor depends on a single parameter: in the DIP case this is fixed at low energies by the experimental constraints on $df(q^2, 0)/dq^2$, while in ref. [8, 9] the free parameter is determined by the OPE, matching the coefficient of the $1/k_E^2$ term for $k_E^2 \to \infty$. In the case of pure electromagnetic decays, such as $\pi^0 \to e^+e^-$ and $\eta \to \ell^+\ell^-$, these two procedures lead to almost equivalent results. On the contrary, in the case of $K_L \to \mu^+\mu^-$ these two approaches lead to a significant numerical difference. Once we assume the convergence condition $f(-k_E^2, -k_E^2) \sim 1/k_E^2$, the largest contribution to the integral (3.3) arises by the low-energy region. For this reason, we believe that the experimental determination of $df(q^2, 0)/dq^2$, at low q^2, provides the most significant constraint on $\chi_{\gamma\gamma}(\mu)$. However, in order to quote a reliable error on this quantity, we need to estimate also the sensitivity of the dispersive integral to the high-energy modes. To this purpose, in the following we shall employ a more general parameterization than the DIP ansatz, but still inspired by the $1/N_C$ expansion, namely

$$\begin{aligned} f(q_1^2, q_2^2) &= 1 + \alpha \left(\frac{q_1^2}{q_1^2 - M_\rho^2} + \frac{q_2^2}{q_2^2 - M_\rho^2} \right) - (1 + 2\alpha - \delta) \frac{q_1^2 q_2^2}{(q_1^2 - M_\rho^2)(q_2^2 - M_\rho^2)} + \\ &\quad + \gamma \left(\frac{q_1^2}{q_1^2 - \Lambda_H^2} + \frac{q_2^2}{q_2^2 - \Lambda_H^2} \right) - (2\gamma + \delta) \frac{q_1^2 q_2^2}{(q_1^2 - \Lambda_H^2)(q_2^2 - \Lambda_H^2)}\,, \end{aligned} \tag{3.6}$$

with $\Lambda_H > M_\rho$. The choice of the four coefficients in (3.6) is such that the sum-rule (3.5) is automatically satisfied and for $\gamma = \delta = 0$ we recover the DIP case (with $\beta = -1 - 2\alpha$).

The expression of $\chi_{\gamma\gamma}(\mu)$ obtained by means of the parameterization (3.6) is

$$\begin{aligned}\chi_{\gamma\gamma}(\mu) &= \frac{3}{2}\ln\left(\frac{\mu^2}{M_\rho^2}\right) - R(M_\rho^2,M_\rho^2) + 2\alpha\left[R(M_\rho^2,0) - R(M_\rho^2,M_\rho^2)\right] + \\ &\quad +\delta\left[R(M_\rho^2,M_\rho^2) - R(\Lambda_H^2,\Lambda_H^2)\right] + 2\gamma\left[R(\Lambda_H^2,0) - R(\Lambda_H^2,\Lambda_H^2)\right], \qquad (3.7)\end{aligned}$$

where, expanding up to the next-to-leading order in the chiral expansion,

$$\begin{aligned}R(M^2,0) &= -\frac{3}{2}\ln\left(\frac{M_\rho^2}{M^2}\right) - \frac{3}{2} + \frac{m_K^2}{M^2}\left[\frac{1-10r_\mu}{6}\ln\left(\frac{m_\mu^2}{M^2}\right) - \frac{11+34r_\mu}{36}\right] + \mathcal{O}\left(\frac{m_K^4}{M^4}\right), \\ R(M^2,M^2) &= -\frac{3}{2}\ln\left(\frac{M_\rho^2}{M^2}\right) - \frac{m_K^2}{M^2}\frac{1-5r_\mu}{3} + \mathcal{O}\left(\frac{m_K^4}{M^4}\right). \qquad (3.8)\end{aligned}$$

The result for $\chi_{\gamma\gamma}(\mu)$ in (3.7) depends on four free parameters: α, γ, δ and Λ_H^2. As anticipated, one combination is fixed by the model-independent constraint following by the experimental measurement of the form factor in the Dalitz decays $K_L \to \gamma\ell^+\ell^-$ and $K_L \to e^+e^-\mu^+\mu^-$. The KTeV collaboration has performed a detailed analysis of this form factor, taking into account the significant distortions of the spectrum induced by radiative corrections [26]. As a result, a coherent determination of the slope

$$\alpha_{\rm exp} = -M_\rho^2 \left.\frac{d}{dq^2}f(q^2,0)\right|_{q^2=0} = \alpha + \frac{M_\rho^2}{\Lambda_H^2}\gamma \qquad (3.9)$$

emerges from all the available channels: $K_L \to \gamma e^+e^-$ ($\alpha_{\rm exp}|_{ee} = -1.63 \pm 0.05$ [5]), $K_L \to \gamma\mu^+\mu^-$ ($\alpha_{\rm exp}|_{\mu\mu} = -1.54 \pm 0.10$ [3]), and $K_L \to e^+e^-\mu^+\mu^-$ ($\alpha_{\rm exp}|_{ee\mu\mu} = -1.59 \pm 0.37$ [4]). Combining these results, we obtain the very precise value $\alpha_{\rm exp} = -1.611 \pm 0.044$. Note that the consistency of the DIP parameterization for the various modes, as well as the one of the BMS model [24], provides a good support in favor of a generic VMD structure, such as the one in eq. (3.6), with a minor role played by the terms associated to the heavier poles.

Taking advantage of the relation (3.9) and expanding up to the first order in the three small ratios m_μ^2/m_K^2, m_K^2/M_ρ^2 and M_ρ^2/Λ_H^2, we can write

$$\begin{aligned}\chi_{\gamma\gamma}(M_\rho) &= -3\alpha_{\rm exp} + \frac{m_K^2}{M_\rho^2}\left[\frac{1}{3} + \alpha_{\rm exp}\frac{1+6\ln(m_\mu^2/M_\rho^2)}{18}\right] - \Delta_\Lambda = (5.83 \pm 0.15) - \Delta_\Lambda, \\ \Delta_\Lambda &= \left[3\gamma\left(1 - \frac{M_\rho^2}{\Lambda_H^2}\right) + \delta\left(\frac{m_K^2}{3M_\rho^2} - \frac{3}{2}\ln\frac{M_\rho^2}{\Lambda_H^2}\right)\right]. \qquad (3.10)\end{aligned}$$

From the expression (3.10) we deduce that: i) $\chi_{\gamma\gamma}(M_\rho)$ shows a rapid convergence in the chiral expansion; ii) the combination of γ, δ and Λ_H^2 which controls the sensitivity to the high-energy modes is Δ_Λ. We also note that $\Delta_\Lambda \ll 1$ in the limit $\Lambda_H \to M_\rho$ (γ and δ are expected to be at most of $\mathcal{O}(1)$ by naturalness arguments), thus the scenario which maximizes the sensitivity to high scales is the case $\Lambda_H^2 \gg M_\rho^2$. Since charm loops play an important role in determining the short-distance behavior of this form factor, we cannot exclude a priori that Λ_H is connected to the charm scale. For this reason, in the following we shall allow Λ_H^2 to reach values up to $10M_\rho^2$.

Estimates of Δ_Λ can be obtained by looking at the $k_E^2 \to \infty$ behavior of $f(-k_E^2, -k_E^2)$:

$$\lim_{k_E^2 \to \infty} f(-k_E^2, -k_E^2) = \frac{2}{k_E^2}\left[(1+\alpha-\delta)M_\rho^2 + (\gamma+\delta)\Lambda_H^2\right] \doteq \mathcal{C}_0 \frac{M_\rho^2}{k_E^2}\,. \tag{3.11}$$

If $|\mathcal{C}_0| \lesssim 1$, as it happens in the case of the pure electromagnetic form factors relevant to $\pi^0(\eta) \to \ell^+\ell^-$ decays [8], we must impose the condition $\gamma \approx -\delta$. As a first qualitative observation, we note that Δ_Λ is rather small once this condition is fulfilled: $|\Delta_\Lambda| < 1$ for $\Lambda_H^2/M_\rho^2 < 10$ and $|\gamma, \delta| < 1$. A more quantitative estimate along this line can be obtained following ref. [9]: bosonizing the partonic currents in (3.1), Greynat and de Rafael argued that $\mathcal{C}_0$ can be computed explicitly to leading order in both the $1/N_C$ and the chiral expansion, obtaining

$$\mathcal{C}_0 = \frac{8\pi^2 F_\pi^2}{N_C M_\rho^2}\left[1 \pm \mathcal{O}(L_5)\right] = 0.38 \pm 0.12 \text{ [9]}\,. \tag{3.12}$$

Using this constraint in (3.11) and imposing the additional conditions $|\gamma, \delta| < 0.5$ and $2 < \Lambda_H^2/M_\rho^2 < 10$, we find $\Delta_\Lambda = 0.3 \pm 0.6$, where the error scales almost linearly with the upper bound on $|\gamma, \delta|$. This result already provides a good support in favor of the smallness of $|\Delta_\Lambda|$; however, we believe that an even more convincing argument can be obtained by looking at the partonic evaluation of $f(q_1^2, q_2^2)$, which we shall discuss in the following.

3.2 High-energy behavior of the $K_L \to \gamma\gamma$ form factor from perturbative QCD

In the euclidean region of large $|q_{1,2}^2|$, with $|q_1^2 - q_2^2| \gg \Lambda_{QCD}^2$, the $K_L \to \gamma\gamma$ form factor can be computed reliably using perturbative QCD. Indeed, this kinematical configuration resembles the case of a heavy-quark decay into a two-body heavy-light system. Here non-perturbative effects associated to the hadronization of the external quark lines have shown to be factorizable — up to power suppressed terms — into appropriate hadronic wave functions [27]. We expect a similar factorization mechanism to hold also for the Green's function in (3.1), where the lowest-order partonic kernel corresponds to the diagrams in figure 2, and the leading hadronic matrix element involved is simply

$$\langle 0|\bar{s}\gamma_\mu\gamma_5 d|K^0(p)\rangle = i\sqrt{2}F_K p^\mu\,. \tag{3.13}$$

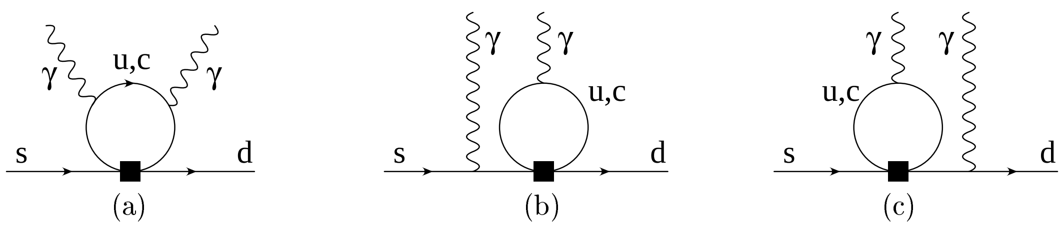

Figure 2: Lowest-order partonic diagrams contributing to the $K_L \to \gamma\gamma$ amplitude.

On general grounds, taking advantage of CKM unitarity, the $\mathcal{O}(G_F\alpha)$ contributions to the $K_L \to \gamma\gamma$ amplitude can be written as

$$\mathcal{A}[K_L \to \gamma\gamma] = V_{us}^* V_{ud} \left[\mathcal{F}_u - \mathcal{F}_c\right] + V_{ts}^* V_{td} \left[\mathcal{F}_t - \mathcal{F}_c\right]. \tag{3.14}$$

As shown explicitly in ref. [20], the impact on the $K_L \to \mu^+\mu^-$ amplitude of the combination proportional to $V_{ts}^* V_{td}$ is completely negligible.[2] Therefore, we can concentrate only on the $[\mathcal{F}_u - \mathcal{F}_c]$ combination, which corresponds to the diagrams in figure 2: this is both UV finite (thanks to the GIM mechanism) and IR safe (for $|q_{1,2}^2| \gg \Lambda_{QCD}^2$). Neglecting power-suppressed terms of $\mathcal{O}(m_u^2/m_c^2)$ and $\mathcal{O}(m_u^2/|q_{1,2}^2|)$ we find

$$\begin{aligned}\mathcal{A}\left[K_L \to \gamma(\varepsilon_1, q_1)\gamma(\varepsilon_2, q_2)\right] &= i\epsilon_{\mu\nu\rho\sigma}\varepsilon_1^\mu \varepsilon_2^\nu q_1^\rho q_2^\sigma \frac{16\alpha_{\rm em} G_F V_{us}^* V_{ud} F_K}{9\pi\sqrt{2}} \left[C_2(\mu) + 3C_1(\mu)\right] \times \\ &\quad \times \left[I\left(\frac{q_1^2}{m_c^2}, \frac{q_2^2}{m_c^2}\right) + T\left(\frac{q_1^2}{m_c^2}\right) + T\left(\frac{q_2^2}{m_c^2}\right)\right]. \end{aligned} \tag{3.15}$$

Here $C_{1,2}(\mu)$ are the Wilson coefficients of the $\Delta S = 1$ effective four-quark hamiltonian, in the notation of ref. [14], and

$$\begin{aligned} I(r_1, r_2) &= \frac{1}{6} + \int_0^1 dx \int_0^{1-x} dy \left\{ \frac{(2-x-y) + (r_1 x + r_2 y)(1-x-y)^2}{1 - (r_1 x + r_2 y)(1-x-y)} - \right. \\ &\qquad \left. - (2 - 3x - 3y) \ln\left[\frac{(r_1 x + r_2 y)(1-x-y) - 1}{(r_1 x + r_2 y)(1-x-y)}\right] \right\}, \\ T(r) &= 2\int_0^1 dx\; x(1-x) \ln\left[\frac{r x(1-x) - 1}{r x(1-x)}\right]. \end{aligned} \tag{3.16}$$

The Wilson coefficients $C_{1,2}(\mu)$, which are understood to be computed at the leading-log level of accuracy at $\mu \sim (q_1^2 + q_2^2)/2$, starting from the initial conditions $C_2(M_W) = 1$ and $C_1(M_W) = 0$, encode all the leading QCD corrections of the type $\mathcal{O}[\alpha_S^n \ln^n(\mu^2/M_W^2)]$. In the kinematical configuration where $M_W^2 \gg |q_{1,2}^2| \gg \Lambda_{QCD}^2$ these are the only potentially large logs which need to be resummed.

In the limit $r_{1,2} \to -\infty$ we find

$$\begin{aligned} I(-r_1, -r_2) &\longrightarrow \frac{\ln[2(r_1 + r_2)]}{r_1 + r_2}\left[1 + \mathcal{O}\left(\frac{(r_1 - r_2)^2}{(r_1 + r_2)^2}\right)\right], \\ T(-r_1) + T(-r_2) &\longrightarrow \frac{1}{r_1 + r_2}\left[1 + \mathcal{O}\left(\frac{(r_1 - r_2)^2}{(r_1 + r_2)^2}\right)\right], \end{aligned} \tag{3.17}$$

which agrees with the results obtained in ref. [1] in the limit $q_1^2 = q_2^2$, and is compatible with the first analysis of this amplitude presented in ref. [18]. On general grounds, the perturbative calculation is not reliable in the limit $q_1^2 = q_2^2$, since this kinematical configuration leads to a soft-momentum kaon. This is indeed one possible criticism which

[2]This conclusion can be qualitatively understood from the smallness of $|V_{ts}^* V_{td}/V_{us}^* V_{ud}|$ and the absence of a hard GIM mechanism ($\mathcal{F}_t \sim m_t^2$) for this amplitude at $\mathcal{O}(G_F\alpha)$.

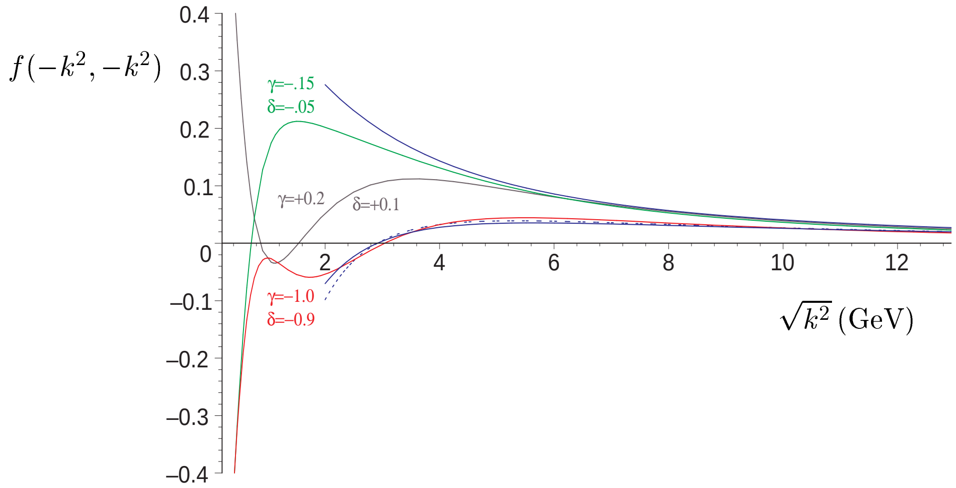

Figure 3: Comparison of the perturbative form factor [lines starting from $\sqrt{k^2} = 2\,\text{GeV}$: upper full curve = no QCD corrections and un-expanded loop functions in eq. (3.16); lower full curve = leading-log QCD corrections included and un-expanded loop functions; lower dotted curve = leading-log QCD corrections included and expanded loop functions in eq. (3.17)] with the phenomenological ansatz, for $\Lambda_H = 3$ GeV and different choices of the parameters (see text).

could be addressed to the DIP analysis. However, by means of eqs. (3.17) we can now show that the limit $q_1^2 = q_2^2$ does not spoil the leading $1/q_{1,2}^2$ behavior in the perturbative region $|q_1^2 + q_2^2| \gg |q_1^2 - q_2^2| \gg \Lambda_{QCD}^2$. For this reason, we shall use the perturbative result in (3.15), in the limit $-q_1^2 = -q_2^2 = k_E^2 \gg \Lambda_{QCD}^2$, to constrain the high-energy behavior of $f(-k_E^2, -k_E^2)$. Note that this type of approach is conceptually rather different from the attempts to evaluate the full $K_L \to \gamma\gamma \to \mu^+\mu^-$ amplitude at the partonic level, as done in previous analyses.

The comparison of the perturbative form factor with the phenomenological parameterization, in the euclidean region, is shown in figure 3. As can be noted, with a suitable choice of parameters it is quite easy to reproduce the perturbative behavior, at large k_E^2, with the phenomenological ansatz. Obviously, the agreement is valid only in a limited range of k_E^2, since the phenomenological parameterization cannot reproduce the logarithmic behaviour of the perturbative result. This is not a problem since the contribution of $f(-k_E^2, -k_E^2)$ to the dispersive integral (3.3) is suppressed for large k_E^2.

In order to define a conservative range of γ, δ and Λ_H compatible with the high-energy behavior, we have imposed the condition that the phenomenological form factor must be in the range defined by the perturbative result, with or without QCD corrections, for $2\text{ GeV} \leq \sqrt{k_E^2} \leq 10\,\text{GeV}$. This condition leaves a considerable uncertainty in the form factor in the region $1\,\text{GeV} \lesssim \sqrt{k_E^2} \lesssim 4\,\text{GeV}$ (see figure 3). This is because the QCD corrections to the effective coupling $[C_2(\mu) + 3C_1(\mu)]$ are sizable at low k_E^2: the partonic result changes even sign with the inclusion of the leading logs [20]. Given the size of these corrections, we believe that the range defined by their inclusion or not is a rather conservative choice.

Despite the uncertainty on $f(-k_E^2, -k_E^2)$ in the region $1\,\mathrm{GeV} \lesssim \sqrt{k_E^2} \lesssim 4\,\mathrm{GeV}$, the resulting range for the weighted integral relevant to the $K_L \to \mu^+\mu^-$ amplitude turns out to be rather limited. This fact should not surprise: it is again a consequence of the fact that the kernel in eq. (3.3) enhances the sensitivity to the low-energy region. As a result of this numerical analysis, we finally obtain

$$|\Delta_\Lambda| \leq 1.0\,. \tag{3.18}$$

in good agreement with the conclusions already derived starting from eqs. (3.11) and (3.12).

3.3 Interference of short- and long-distance components

As can be noted from figure 3, the short-distance constraint on $f(-k_E^2, -k_E^2)$ is not sufficient to determine the sign of the physical $K_L \to \gamma\gamma$ amplitude, or the sign of the coupling $\mathcal{N}$ in eq. (3.2). By construction, we have assumed $\mathcal{N} > 0$ in the phenomenological analysis, hiding this ambiguity in the sign of κ, the parameter of eq. (2.10) which controls the interference between $\chi_{\gamma\gamma}(\mu)$ and $\chi_{\rm short}$.

Within Chiral Perturbation Theory, the leading contributions to the on-shell $K_L \to \gamma\gamma$ amplitude are the tree-level pole diagrams with π^0, η and η' exchange[3] [31]. In all realistic scenarios of pseudoscalar meson mixing, the contributions induced by η and η' poles cancel to a large extent and the $K_L \to \gamma\gamma$ amplitude turns out to be dominated by the π^0 pole (see e.g. ref. [7, 10, 31]). The π^0 pole contribution to the $K_L \to \gamma\gamma$ amplitude can be written as

$$\mathcal{A}^{(\pi^0)}(K_L \to \gamma\gamma) = \frac{i2G_8 F_\pi \alpha_{\rm em}}{\pi} \frac{m_K^2}{m_K^2 - m_\pi^2} \epsilon_{\mu\nu\rho\sigma} \varepsilon_1^\mu \varepsilon_2^\nu q_1^\rho q_2^\sigma\,, \tag{3.19}$$

where G_8 denotes the leading $(8_L, 1_R)$ coupling of the chiral non-leptonic weak lagrangian: $\mathcal{L}_W^{(2)} = G_8 {\rm tr}(\lambda_6 \partial U^\dagger \partial U)$ [31]. The sign of G_8 cannot be determined in a model-independent way; however, it can be predicted by the $\Delta S = 1$ partonic lagrangian computing the hadronic matrix elements of four-quark operators in the large N_C limit (or employing naive factorization). In the convention defined by eq. (2.9), this procedure leads to ${\rm sgn}(G_8) < 0$ [28, 29], which would imply a *negative* interference between $\chi_{\gamma\gamma}(\mu)$ and $\chi_{\rm short}$. In other words, under the following two assumptions:

i. ${\rm sgn}[\mathcal{A}(K_L \to \gamma\gamma)] = {\rm sgn}[\mathcal{A}(K_L \to \pi^0 \to \gamma\gamma)]$;

ii. ${\rm sgn}[\langle \pi^0 | \mathcal{H}_W | K_L \rangle] = {\rm sgn}[\sum C_i(\mu) \langle \pi^0 | O_i(\mu) | K_L \rangle_{N_C \to \infty}]$;

we need to set $\kappa < 0$. This conclusion agrees with the result of ref. [10], where a negative interference between $\chi_{\gamma\gamma}(\mu)$ and $\chi_{\rm short}^{\rm SM}$ has also been derived employing large N_C arguments. However, it is worth to emphasize that this conclusion depends only on the two assumptions stated above, whose validity go beyond the $1/N_C$ expansion.

[3]The η' exchange, formally of higher order, cannot be neglected due to the cancellation of the lowest-order π^0 and η diagrams in the limit where we neglect η–η' mixing and apply the Gell-Mann–Okubo mass formula.

JHEP01(2004)009

4. Short-distance constraints from $K_L \to \mu^+\mu^-$

Taking into account the estimate of Δ_Λ in (3.18), we can quote as final estimate for the low-energy coupling $\chi_{\gamma\gamma}$:

$$\chi_{\gamma\gamma}(M_\rho) = 5.83 \pm 0.15_{\rm exp.} \pm 1.0_{\rm th.} \tag{4.1}$$

As expected, this result is numerically rather close to the DIP one [1], and to the one of ref. [10], while it is substantially different from the value $\chi_{\gamma\gamma}(M_\rho) = 2.18 \pm 0.15 \pm 0.9$ quoted in ref. [9]. As shown in the previous section, the difference between our result and ref. [9] does not arise by the high-energy behavior of the $K_L \to \gamma\gamma$ form factor, which is essentially the same in both cases, but it is a consequence of the inclusion of the low-energy condition (3.9).

Although the final estimate in (4.1) is numerically rather similar to the one of ref. [1], the present analysis is certainly more conservative. Indeed, the error in (4.1) is mainly of theoretical nature and should be regarded as the definition of a flat confidence interval, rather than the standard deviation of a statistical distribution. It will be very hard to decrease this error only with the help of new experimental data on $K_L \to \gamma\ell^+\ell^-$ and $K_L \to e^+e^-\mu^+\mu^-$ decays: the only significant improvement could arise by the determination of the quadratic slope β from $K_L \to e^+e^+\mu^+\mu^-$, which is beyond the reach of present kaon facilities. On the other hand, a significant step forward could in principle be obtained by means of Lattice-QCD calculations of the form factor in the region $1\,\text{GeV} \lesssim \sqrt{k_E^2} \lesssim 4\,\text{GeV}$.

Using the result (4.1) in eq. (2.8), and solving the quadratic equation in terms of $\chi_{\rm short}$, we finally obtain

$$\chi_{\rm short} = \begin{cases} -1.7 \pm 1.4 \\ \;\;\,0.3 \pm 1.4 \end{cases} \qquad \longrightarrow \qquad -3.1 < \chi_{\rm short} < 1.7\,. \tag{4.2}$$

In the most conservative scenario, i.e. without assumptions about the sign of the interference between short and long-distance amplitudes, only the upper bound on $|\chi_{\rm short}|$ can be used. From the latter we derive the conservative upper bound

$$B(K_L \to \mu^+\mu^-)_{\rm short} < 2.5 \times 10^{-9}\,. \tag{4.3}$$

It is worth to emphasize that the result in (4.2) is perfectly compatible with the SM also when the sign estimate discussed in the previous section is taken into account: $\chi_{\rm short}^{\rm SM} = -1.9 \pm 0.2$ (for $\kappa < 0$). Using the SM expression (2.10), the bound (4.2) can be translated into the following range for $\bar\rho$:

$$-0.5 < \bar\rho < 2.1 \;\; (2.9)\,, \tag{4.4}$$

where the number between brackets does not take into account the sign estimate.

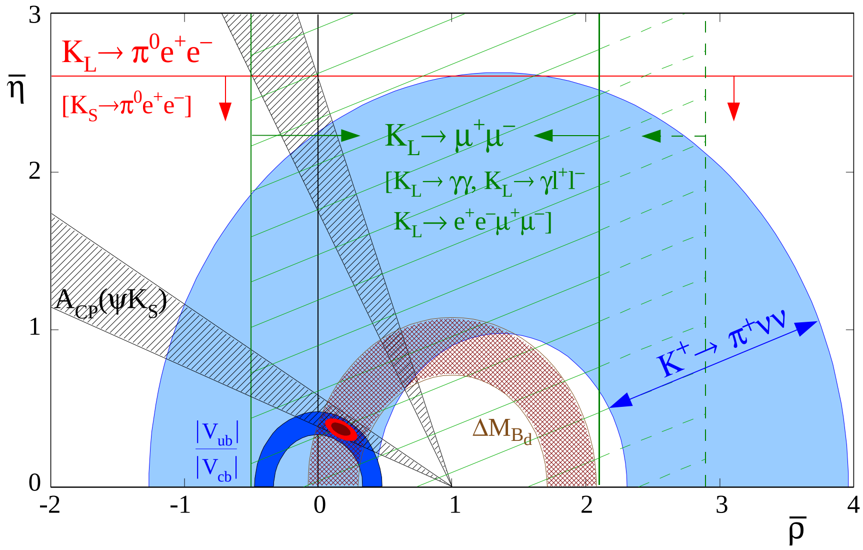

Figure 4: Summary of the present constraints in the $\bar{\rho}$–$\bar{\eta}$ plane from rare K decays: the $K_L \to \mu^+\mu^-$ bound on $\bar{\rho}$ discussed in this work (with and without the sign estimate); the $K_L \to \pi^0 e^+ e^-$ bound on $\bar{\eta}$ [29] and the $K^+ \to \pi^+ \nu\bar{\nu}$ ellipse [30]. The constraints on $|V_{ub}|$ and B–$\bar{B}$ mixing, from ref. [15], are also shown for comparison.

As can be seen in figure 4, this bound does not compete in precision with present CKM constraints from B physics; however, it does compete with constraints from other $\Delta S = 1$ FCNC processes, such as $K^+ \to \pi^+ \nu\bar{\nu}$. The interest of these bounds is their implications for non-standard scenarios. For instance, using eq. (2.12), the $K_L \to \mu^+\mu^-$ bound leads to

$$-2.2\ (-3.3) < \frac{\mathrm{Re}\,(Z_{ds})}{\mathrm{Re}\,(Z_{ds}^{\mathrm{SM}})} < 2.2\,, \tag{4.5}$$

which is one of the most stringent constraints on possible non-standard FCNC couplings of the Z boson.

5. $K_S \to \mu^+\mu^-$

The most general decomposition of K^0 $(\bar{K}^0) \to \ell^+\ell^-$ decay amplitudes does not include only the s-wave (A) component in eq. (2.1), but also a p-wave (B) term:

$$\mathcal{A}(K^0 \to \ell^+\ell^-) = \bar{u}_\ell (iB + A\gamma_5) v_\ell\,, \tag{5.1}$$

with a corresponding decay rate given by [31]

$$\Gamma(K_{L,S} \to \ell^+\ell^-) = \frac{m_K \beta_\ell}{8\pi}\left(|A|^2 + \beta_\ell^2 |B|^2\right), \qquad \beta_\ell = \left(1 - \frac{4m_\ell^2}{m_K^2}\right)^{1/2}. \tag{5.2}$$

The two amplitudes have opposite CP, such that CP-conserving contributions to K_L and K_S decays are generated by A and B, respectively. Long-distance contributions generated by intermediate two-photon states can lead to both A and B amplitudes, but in both

JHEP01(2004)009

cases these have a negligible CP-violating component. Short-distance contributions of the SM type can contribute only to the A amplitude, but in this case CP-violating phases are expected to be $\mathcal{O}(1)$. For this reason, within the SM and in any SM extension with the same basis of effective FCNC operators, we can safely neglect the B term in the $K_L \to \mu^+\mu^-$ amplitude (as we have done so far). On the other hand, in the $K_S \to \mu^+\mu^-$ case we need to keep both types of amplitudes and we can write

$$\Gamma(K_S \to \ell^+\ell^-) = \frac{m_K \beta_\ell}{8\pi} \left[(\mathrm{Im}\, A_{\mathrm{short}})^2 + \beta_\ell^2 (\mathrm{Re}\, B_{\gamma\gamma})^2 + \beta_\ell^2 (\mathrm{Im}\, B_{\gamma\gamma})^2 \right] . \tag{5.3}$$

The remarkable feature of eq. (2.1) is the fact that the three contributions add incoherently in the total rate. It is then much simpler to derive constraints on the short-distance component from the experimental limit on $\Gamma(K_S \to \ell^+\ell^-)$: in the most conservative case, we can derive a model-independent bound on $|\,\mathrm{Im}\, A_{\mathrm{short}}|$ simply setting to zero the long-distance terms.

Analogously to eq. (2.9), within the SM one finds

$$\mathrm{Im}\, A_{\mathrm{short}}^{\mathrm{SM}} = -\frac{G_F \alpha_{\mathrm{em}}(M_Z)}{\pi \sin^2\theta_W} \sqrt{2} m_\mu F_K \,\mathrm{Im}(V_{ts}^* V_{td}) Y(x_t) \tag{5.4}$$

which leads to

$$\begin{aligned} B(K_S \to \mu^+\mu^-)_{\mathrm{short}}^{\mathrm{SM}} &= 1.0 \times 10^{-5} \times |\mathrm{Im}(V_{ts}^* V_{td})|^2 \\ &= 1.4 \times 10^{-12} \times \left| \frac{V_{cb}}{0.041} \right|^4 \times \left| \frac{\lambda}{0.223} \right|^2 \times \bar{\eta}^2 . \end{aligned} \tag{5.5}$$

According to the value of $\bar{\eta}$ obtained from global CKM fits, this contribution is in the 10^{-13} range. However, the present constraints on $\bar{\eta}$ derived only from $\Delta S = 1$ FCNC processes are rather weak (see figure 4): this implies that, at present, new-physics scenarios where $B(K_S \to \mu^+\mu^-)_{\mathrm{short}}$ reaches the 10^{-11} level are perfectly allowed.

Contrary to the $K_L \to \mu^+\mu^-$ case, the dispersive long-distance $K_S \to \mu^+\mu^-$ amplitude is unambiguously determined at the lowest order in the chiral expansion: it arises by two-loop diagrams of the type $K_S \to \pi^+\pi^- \to \gamma\gamma \to \mu^+\mu^-$, which are finite due to the absence of corresponding local terms [12]. The dispersive amplitude is about 2.3 times the absorptive one, and summing the two contributions Ecker and Pich found [12]:

$$B(K_S \to \mu^+\mu^-)_{\gamma\gamma} = 1.9 \times 10^{-6} \times B(K_S \to \gamma\gamma) \approx 5 \times 10^{-12} . \tag{5.6}$$

Due to possible higher-order chiral corrections, this prediction is expected to hold within a 30% error.

Comparing eq. (5.5) and eq. (5.6), we conclude that a search for $K_S \to \mu^+\mu^-$ in the 10^{-11} range would be very useful. An evidence of $K_S \to \mu^+\mu^-$ well above 10^{-11} would be a clear signal of new physics. In absence of a signal, as expected within the SM, bounds on $B(K_S \to \mu^+\mu^-)$ close to 10^{-11} could be translated into interesting model-independent bounds on the CP-violating phase of the $s \to d\ell^+\ell^-$ amplitude.[4]

[4]These bounds could be very useful to discriminate among new-physics scenarios if other modes, such as $K^+ \to \pi^+\nu\bar{\nu}$, would indicate a non-standard enhancement of the $s \to d\bar{\ell}\ell$ transition

6. Conclusions

The short-distance dominated $\Delta S = 1$ FCNC transitions are a key element to investigate the mechanism of quark-flavor mixing, and the rare decays $K_{L,S} \to \mu^+\mu^-$ represent one of the most useful experimental probes of these processes. In this paper we have re-analyzed the extraction of short-distance constraints from $K_L \to \mu^+\mu^-$. To this purpose, we have quantified the uncertainty of the long-distance $K_L \to \gamma\gamma \to \mu^+\mu^-$ amplitude. The latter has been evaluated employing a semi-phenomenological approach to the $K_L \to \gamma\gamma$ form factor. At short distances, this approach is essentially equivalent to the one of ref. [8, 9]; however, the semi-phenomenological approach is superior in the low-energy region, where it takes into account the precise experimental constraints on the Dalitz modes. As we have shown, this region provides the dominant contribution to the dispersive integral needed to evaluate the $K_L \to \gamma\gamma \to \mu^+\mu^-$ amplitude.

The short-distance bounds thus derived, expressed as effective bounds on the CKM parameter $\bar{\rho}$, are summarized in figure 4. Although not competitive with B-physics constraints, these bounds provide a serious challenge to many NP models. The present range is mainly determined by the theoretical error of the approach and it will be very hard to decrease it by means of experimental data only. A minor improvement could be expected with better data on the ratio $\Gamma(K_L \to \mu^+\mu^-)/\Gamma(K_L \to \pi^+\pi^-)$ and thus a smaller error in eq. (2.8). As we have stressed, a major improvement could be obtained by means of Lattice-QCD calculations of the $K_L \to \gamma\gamma$ form factor in the euclidean region.

Finally, we have briefly discussed the possibility to extract short-distance constrains also from $K_S \to \mu^+\mu^-$. As we have shown, bounds on $B(K_S \to \mu^+\mu^-)$ close to 10^{-11} could be translated into interesting model-independent bounds on the CP-violating phase of the $s \to d\ell^+\ell^-$ amplitude.

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